GOL 2021: International Conference
Applied Category Theory Graph-Operad-Logic
Professor Zbigniew Oziewicz in memoriam

Book of Abstracts

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Zbigniew Oziewicz was a promoter of Category Theory and its applications to every scientific field. He gave special attention to fundamental problems in physics like Relativity Theory, Electromagnetics, Quantum Mechanics, Elementary Particles Theory, among others. He organized many seminars, conferences, congress, around those scientific fields. One of them was the "Applied Category Theory, Graph - Operad - Logic" international workshop series. This GOL 2021 International Conference is dedicated to Professor Zbigniew Oziewicz, in memoriam.

Session 1

Influence and inspiration, Zbigniew Oziewicz in Mexico

1.1 Remembering 25 years of activities with Professor Zbigniew Oziewicz in Mexico

Larissa Sbitneva

My communication will rather concern to the personality of Professor Oziewicz, leaving the account of his deep and broad contributions to different brunches of Science for his numerous colleagues from all over the world. We were in rather permanent communications for almost 25 years, where he served as a leading professor at the scientific seminars at FESC, organizing International Conferences, teaching courses and directing research for thesis. I prefer to make an emphasis on his pedagogical performance during teaching the general courses as well as his influence and inspiration in formation of future researches, although our mathematical interactions concerned mostly to non-associative algebraic structures associated to the Relativity were culminated during the period of September–December 2020 when I had a privilege to be invited as a participant of the scientific seminars conducted by Professor Oziewicz on my leave of absence. Zbigniew was extremely open in human relationships with so many people around him: once his elegant figure, always in almost white suits, appeared on the university pathways, different persons approached him to express their greeting, to whom he always had something personal and intimate to say and treat them with the very special gentle manners of his only European style expressing the essential benevolence of his attitude towards everyone around him. His scientific seminars attracted also his graduated students as well the colleagues from other faculties, his lectures and conversations were widely appreciated, since not only the scientific questions and controversies were explained with all detailed but also the historical context was presented with his very human personal attitude. In the classroom during the breaks he was always surrounded by students showing their admiration and affection. He was tireless in explaining his understanding of laws of the Universe as four dimensional Space-time, although, as he confessed sometimes in his scientific seminars, not so many specialists were ready to share his vision. I am sure that many of his students and collaborators remember the marvelous programs of the conferences, especially the Eighth International Workshop Applied Category Theory Graph-Operad-Logic, 2010 - Nayarit, Mexico, en la Universidad Autónoma de Nayarit.

1.2 Zbigniew Oziewicz: A journey through fundamental problems in physics

Jesus Cruz Guzman

The GOL 2021 International Conference "Applied Category Theory Graph-Operad-Logic", Professor Zbigniew Oziewicz in memoriam, is an opportunity to reminisce about some of his ideas that many of us, who knew and worked with him, have embraced. During his almost 30 years at FESC UNAM, he touched many scientific fields. Allow me to walk you through some of the subjects that I had the opportunity to undertake with him. One quests to pursue was: How to teach mathematics and physics as opposed to the conventional way, that is based on wrong fundamentals and nonetheless, is followed in almost the entire world? Mathematics was always his starting point that, based on Category Theory, he tried to apply to everything. Here I am going to present some examples, starting with the approach of category theory as a universal language, where the concepts of operad and multicategory play an important role. His ideas about differential calculus were a principal topic with the approach of calculus as a Lie graded algebra of derivations of the Grassmann algebra of differential forms. His applications to Classical Mechanics, Electromagnetism, Quantum Mechanics, among many more, were remarkable. In those applications the physical system is subject to universal laws, absolute laws because they are observer independent. The empirical laws (observer dependent) are then expressed by an algebra of idempotents. An important subject was his ideas about Special Relativity where relative velocity was a morphism in a Groupoid Category.

1.3 Benefits of using the virtual classroom for the training process of students to promote their interest in science and research

Hilda María Colín-García, Larissa Sbitneva and Zbigniew Oziewicz

Our proposal raises a double objective, on the one hand, to present the experience of Professor Zbigniew Oziewicz in the use of the Moodle virtual classroom, to promote interest in science in students and, on the other, to demonstrate the benefits that this tool can have in favor of teaching, learning and research. The work describes the experiences and academic dynamics developed by Professor

Oziewicz using the virtual classrooms of the UNAM. The Moodle virtual classroom is claimed to be an excellent pedagogical tool for teaching and learning basic concepts of mathematics, philosophy, epistemology, physics, and the vision of the universe. At UNAM there are groups of 40 to 50 students and in face-to-face or synchronous sessions, it is practically impossible for everyone to express their individual opinions and ideas. Thus, the virtual classroom allows the design of weekly work dynamics, in which the student, with the teacher's advice, investigates different sources of information such as digital libraries, textbooks, Wikipedia, at least three different definitions of basic concepts of the subject in which you can contrast, the definition of the teacher, those of other investigated authors and create your own. The premise holds that understanding is individual and should not necessarily be valid for a group as a whole. The teacher must motivate and encourage students to reflect, investigate and formulate their own thesis or proposals. It is affirmed that students have the right to have their own learning and freedom to express their individual opinions, not necessarily shared by other students or even by the teacher himself. All of this is delivered individually through the virtual classroom with weekly feedback. On this basis, each week the work developed by the students is commented on in a group, it is discussed, reflected, comments and reflection is motivated and not memorization.

Session 2

Category theory, graphs, logic

2.1 Calculus and Non-Commutative Worlds

Louis H. Kauffman

There are many reasons for reformulating calculus in a non-commutative framework. Once this is done one defines derivatives as commutators. Thus one may write $DF = FN - NF = [F, N]$ where N is a suitable representative for that derivative. The simplest and most fundamental reason for doing this is that discrete calculus can be formulated in this way. The talk will begin by explaining this fact of mathematical life. Discrete derivatives are actually all we have prior to the fantasy of limits that makes for the context of continuity and classical calculus. Derivatives as commutators seldom commute with one another. Derivatives that do not commute with one another are the signals of curvature in places were spaces have not yet appeared. By starting in this place of the non-commutative algebra of differences, we find that a remarkable body of the formalism of physics emerges naturally from the pen that writes the noncommutative calculus. In particular we see how the patterns of gauge theory appear and how they begin to be related with relativity and quantum theory. In this talk we will go back to Hermann Weyl's original idea to unify gravity and electromagnetism that led him to discover gauge theory and see how it is related to non-commutative worlds and with the relationship of these worlds to quantum theory. Please note that the author is not offering a theory of everything. He is offering a mathematical starting place for everything. Shouldn't that be compared with foundational enterprises such as Category Theory or the Theory of Sets or the possibilities of a Distinction? Of course it should. It is hoped that this talk can elucidate some of that as well.

2.2 A Philosophical Honing of Quantum Natural Language Processing (QNLP)

Carlos César Jiménez

By QNLP, Coecke et. al mean a compositional processing of "natural language" on quantum hardware matching the way in which quantum systems compose. Crucial for the aforementioned implementation has been the Categorical Distributional Compositional model for natural language (DisCoCat), a mathematical framework that unifies the distributional theory of meaning in terms of vector space models, and a compositional theory for grammatical types. However, both DisCoCat and QNLP can still be honed within a nuanced broader philosophy of language background. Our proposal aims at providing this contextualization by recalling several arguments put forward in the previous decades by the Dutch logician and philosopher Martin Stokhof regarding the challenges of devising formal systems in order to deal with natural language. Hopefully, this might help broadening the scope of QNLP to succesfully meet the discursive, dialogical and hermeneutical challenges of our diversified linguistic practices from a theoretical and experimental point of view.

2.3 Sobre Independencia Axiomática de una Lógica tres-valuada

Miguel Pérez-Gaspar and Everardo Bárcenas

La lógica $CG'3$ es una Lógica tres-valuada y paraconsistente cuyos valores de verdad están en el dominio $D = \{0, 1, 2\}$, el conjunto de valores de verdad designados es $D* = \{1, 2\}$ y cuyos conectivos son: $\wedge, \vee, \rightarrow$ y$\neg$. Un subconjunto Y del conjunto de axiomas de una teoría se dice que es independiente si alguna fórmula bien formada en Y no puede ser demostrada por medio de las reglas de inferencia del conjunto de aquellos axiomas que no están en Y. En este trabajo se estudia la independencia axiomática de la lógica $CG'3$.

2.4 Multigraphs of multigraphs: modeling hierarchical structures and scale-change

Fernando R. Velázquez Quesada

A (directed) graph is a mathematical structure used for modeling morphisms (understood here as abstract binary relations) between objects. As useful as it is, sometimes one is interested in modelling not morphisms between abstract objects, but rather morphisms between structures that grow more and more complex. This manuscript, based on the lecture notes "Gráficas de Gráficas. Una breve introducción a teorıía de categorías", discusses a generalisation to the notion of a graph, first moving up to an $n - graph$ (the well-known multigraph), and then reaching an $(n1, ..., nk) - graph$ (a multigraph of multigraphs). With these tools, it is possible to model not only hierarchical structures (as a multigraph does, by allowing morphisms between objects, morphisms between morphisms over objects, and so on) but also scale-change (by allowing morphisms over objects with further internal structure, as a morphism between $1 - graphs$). A multigraph of multigraphs has pedagogical advantages. First, the structure is built in a natural way. Second, by requiring specific morphisms (for identity and partial composition) with particular behaviour (the composition is associative, and the identity can be composed), it allows a step-wise construction of a category, function and natural transformation, the basic notions in category theory.

2.5 Mathematics and physical meaning in relativistic quantum mechanics

Peter Rowlands

The mathematics used to code physical theories is not neutral, but frequently contributes to their meaning. This concept is examined in the case of the algebra used in relativistic quantum mechanics, where successive changes lead to increased physical meaning and more powerful physical analysis. Beginning with a mathematical structure which is supposedly used for operational 'convenience', we use a progressive replacement strategy to provide a series of physical explanations and previously unrealised physical consequences.

Session 3

Theory of relativity

3.1 Concepts of Relative Acceleration

Bill Page

A material object may be considered a process whose defining characteristic is the flow of time in a manifold of events. This flow may be represented by a vector field but does not in itself describe motion. All motion is relative, i.e., a collective property of two or more objects. Denoting the motion of object B relative to object A by w_{AB}, the assumption that the composition of motions is associative $w_{AB} \circ w_{BC} = w_{AC}$ implies that the motion of a system of objects is completely determined by the motion of pairs of objects alone. w_{AB} is unique and completely defined in terms of the algebraic properties of process vectors of A and B. Each motion has an inverse $w_{AB}^{-1} = w_{BA}$ but it need not be additive (reciprocal), i.e., $w_{AB} \circ w_{BA} = w_{AA} \neq 0$. One aspect of motion is represented by the relative velocity vector v_{AB} in the spatial kernel of process A. v_{AB} is central to the theory of relativity. Another aspect of motion is represented by the relative acceleration u_{AB} but acceleration is more controversial. In contrast to the usual definition of acceleration in relativity, Zbigniew Oziewicz defined relative acceleration in terms of the Lie bracket of the process vectors which turns out not to be confined to the kernel of A. In this paper we compare the conventional relativistic definitions of velocity and acceleration with that of Oziewicz.

3.2 Evolution equations of relativistic dynamics along geodesic lines of hyperbolic and elliptic spaces

Robert Yamaleev

The mass-shell equation is treated as an characteristic equation of the second order ordinary differential equation. The second order differential equation is transformed into the Riccati-type equation describing the relativistic dynamics as an evolution motion along the geodesic lines with respect to evolution parameters identified with the lengths of the geodesic lines in hyperbolic and elliptic geometries. The relativistic velocity is defined by derivatives of the lengths of geodesic lines. The theory is worked out by taking advantage from the generalized complex algebra and the connected trigonometry.

3.3 Relative binary, ternary and pseudo-binary 4D velocities in the Special Relativity

Grzegorz M. Koczan

Zbigniew Oziewicz was a pioneer of the 4D space-time approach to covariant relative velocities. In 1988 (according to private correspondence) he discovered two types of 4D relative velocities: binary and ternary, along with the rules for adding them. They were first published in conference materials in 2004, and the second time in a peer-reviewed journal in 2007. These physically logical and mathematically precise concepts are so subtle that Oziewicz's numerous preprints have yet to receive the recognition they deserve. The review part of the article presents the Oziewicz–Bolós binary velocity and the Oziewicz–Dragan ternary velocity. The Einstein–Oziewicz velocity, which is a four-dimensional generalization of Einstein velocity addition, also has a ternary character. The original part of the work introduces two ternary generalizations of relative binary velocity called pseudo-binary relative velocities: cross and axial. The first pseudo-binary cross velocity is a proper 4D generalization of the author's 3D jet velocity (3D binary velocity). The second is a 4D generalization of the 3D axial velocity of Fernándeuz-Guasti and the author. The cross velocity is a relatively simple modification of the binary velocity, while the second pseudo-binary axial velocity is a bit more complicated – almost like typical ternary velocity.

3.4 The Space-Time Theories Exploratorium

Mariana Espinosa-Aldama

Interactive concept lattices for some fifty different theories of space-time and gravitation can be explored at remo.cua.uam.mx/vis/Exploratorium. The lattices relate concepts following partial ordering and a formal concept analysis and are visualized in the web page with D3. The formal contexts were built according to several formal classifications and reconstructions under the Lagrangian formalism, coordinate independent in 4D manifolds. Attributes represent groups of axioms and are color coded according to their function as geometrical objects, mathematical conditions, typifications and constrictions, actions, equations of motion and field equations. Clicking on nodes will show super-concepts (foundations) and subconcepts (specialized models) of an actual model, following Balzer, Moulines and Sneed's structuralist semantic set theoretic school of philosophy of science. The interactive network visualizations allow the user to appreciate the formal relations between theories, their common concepts and foundations, to identify groups of theories and identify some patterns of theory evolution. Classifications of classic physical theories, space-time theories, metric gravitational theories, f(R) theories, MoNDian theories and metric affine theories are represented in groups. A "holon" of theories comprises most of these groups showing a temporal progress from left to right as concepts evolve or are added in time, as well as locating topological and geometrical concepts in the upper part, while physical and more specialized models are located in the lower part of the lattice. Appreciating such patterns may provide a sense of understanding of the general components of such field of knowledge. The Space-time Theories Exploratorium is a useful tool for science communicators, educators, students and public interested on the foundations of physics. It is also a useful map for researchers studying alternative theories of gravitation as it allows them to get a sense of where they stand in a sea of theories. The FCA methodology can be applied, following the structuralist program, in other sciences that may be formally axiomatized.

3.5 Classical Relativity and Quantum Relativity versus Special Relativity

Romuald Brazis

The trinity of time, position and motion, as it is apprehended in the Relativity concepts, was the subject of my continuous discussions with Professor Dr Zbigniew Oziewicz since 2003. We have met in 2003 at the Polish-Mexican workshop on Mathematics in Bedlewo near Poznań. Zbigniew Oziewicz presented there a topic on the Special Relativity noticing that the Lorentz transformations that are basic for the Special Relativity do not meet the requirements of a group. This deep remark moved me to look at the Special Relativity from the new perspective, which was not even mentioned in my students' times. I started from a review of the issues of space, time and motion in the textbooks of Physics for the schools and universities in Lithuania and Poland, and wrote a couple of papers on it. Professor Zbigniew Oziewicz found the papers very interesting, encouraged me to continue the research, and opposed some my considerations, as he was convinced that the opposition is the only driving force of science. Zbigniew Oziewicz has gone, unfortunately, leaving me with my new arguments suspended in an "empty space". The Workshop in memoriam of Zbigniew Oziewicz opens the best way to make his risen ideas alive. In the details of my report I'd like to discuss the Classical Relativity that arises from the Aristotle's Physik and includes the concepts of Copernicus, Galileo, Romer, Bradley and Doppler, the Quantum Relativity with its well-balanced particle-wave concepts of Newton, Bradley, Doppler, Umow, Bartoli, Lebedew and Planck, and the Special Relativity, which with its a priori imposed Lorentz factor does not agree neither with the Classical nor the Quantum Relativity.

Session 4

Noncommutative Geometry, Differential Calculus

4.1 Noncommutative Differential Calculus - 25 years later

Andrzej Borowiec

In the first part of my presentation, I revisit some results obtained in the early nineties in collaboration with Slava Kharchenko and Zbigniew Oziewicz. We were focused on algebraic aspects of differential calculus (in terms of an exterior algebra of differential forms) on noncommutative (=quantum) spaces. I highlight also some more recent developments concerning covariance, quantum group differential calculus, the notion of noncommutative vector fields and their applications to Quantum Riemannian Geometry by E. Beggs and S. Majid.

4.2 Some Aspects of Zbigniew Oziewicz's Interpretation of Classical Thermodynamics

Dalia Cervantes, Hilda Maria Colín and Jesus Cruz

Professor Zbigniew Oziewicz was a strong critic of the science teaching models followed by the universities around the world. In this work, we present some parts of Oziewicz understanding on classical thermodynamics, in which the scalar ring is an essential constituent. Concepts like temperature, entropy, internal energy among others are elements of the thermodynamic scalar ring (SR), then exist levels associated with thermodynamic states. For Professor Oziewicz the thermodynamic processes are (directional) derivations on SR, then each scalar has many possible (directional) derivations, included in these kinds of processes the equilibrium are remarkable. One example of them are isothermal processes in which the derivation of temperature in the direction of the isotherms is the zero real number. Furthermore there are scalar subrings conserved by each process. Using the Ricci Theorem (1884), Oziewicz showed that processes are defined independently of the selection of coordinates. This free choice is fundamental to the Professor's point of view of science. Finally with the application of Ricci's Theorem to thermodynamic processes as an expression among partial derivatives emerges easily in contrast to the traditional presentation.

4.3 Non-commutative Differential Calculus and its Applications

Victor Abramov

We propose a notion of (q, σ, τ)-differential graded algebra, which generalizes the notions of (σ, τ)-differential graded algebra and q-differential graded algebra, where q is an Nth primitive root of unity. We construct two examples of (q, σ, τ)-differential graded algebra, where the first one is constructed by means of a generalized Clifford algebra with two generators (reduced quantum plane) and our construction is based on a (σ, τ)-twisted graded q-commutator. In order to construct the second example, we introduce a notion of (σ, τ)-pre-cosimplicial algebra.

4.4 Leibniz algebras and differential graded Lie algebras

Jacob Mostovoy

Leibniz algebras are a non-antisymmetric version of the Lie algebras. I will describe how they appear in the context of differential graded Lie algebras and how this connection explains many facts (maybe, every fact) about the Leibniz algebras.

Session 5

Abstract Algebra, Non-commutative Calculus

5.1 Disruptive Foundations of Mathematical Analysis

Jose G. Vargas

We deal with implications of partial derivatives having mutually incompatible defining conditions, i.e. what remains constant in the differentiation with respect to each variable. Erich Kähler dealt with this issue for $A(\equiv a_i(x)\frac{\partial}{\partial x_i})$ by reducing it to a single partial derivative, $\frac{d}{d\lambda}$. For this purpose, he derived coordinate systems $(y_i)(i = 1, \ldots, n)$ where y_n is taken to be the independent variable λ in $\frac{dx_i}{d\lambda} = a_i(x)$ as input, and the other coordinates are first integrals. The inequality $Au = a_i(x)\frac{\partial u}{\partial x_i}$ emerges. The said incompatibility is obviously behind this unexpected result, also confirmed by application to differential 1-forms of Laurent Schwarz'es method for obtaining primitives of closed differential forms. Consider next the operator $d(\equiv dx_i\frac{\partial}{\partial x_i})$. Elie Cartan suggested the extension of the concept of exterior derivatives to include non-differentiable ones. One faces the riddle that, whereas the commutativity of partial derivatives follows from the square of $dx_i\frac{\partial}{\partial x_i}$ being zero, such commutativity requires the satisfaction of in depth, idiosynchratic conditions, as Schwarz has shown. Kahler's obtaining of the right expression for Au brings with it the concept of good and bad coordinate systems. It emerges that the bad ones are those for which non-commuting partial derivatives result. This speaks of the desirability of having a concept of differentiable manifold more discriminating than the usual one. Finally, the expression that Kähler finds for Au is, for differential 1-forms, the same as the expression for the Lie derivative of a vector field with the same components as u. This speaks of the unsuitability of considering partial derivatives as vector fields. Related to this, differential forms are, as in Arfken's book on analysis, integrands rather than antisymmetric multilinear functions of vectors. The inter relation of these topics makes them reach deeply into the foundations of mathematical analysis.

5.2 Kepler problem and Jordan Algebra

Guowu Meng

We would like to report that the mathematical secret of the Kepler problem for planet motion lies in the algebra invented by Ernst Pascual Jordan in the reformulation of quantum mechanics.

5.3 Recent results on the Theory of binary Lie Algebras

Liudmila Sabinina

Nowadays the tangent algebras of smooth loops are called Sabinin algebras. We will present a survey on the theory of Binary Lie algebras and Malcev algebras, which are important cases of Sabinin algebras, as they are tangent algebras of smooth diassociative and Moufang loops correspondently. In our talk we will discuss some interesting properties of Binary Lie algebras and particularly of Malcev algebras with the identity $J(x; y; zt) = 0$. Open questions on the area will be considered.

5.4 On the relative finite dimension in modules over rings

Tapatee Sahoo, Harikrishnan Panackal, Babushri Srinivas Kedukodi and Syam Prasad Kuncham

It is well-known that the dimension of the vector space over a field is a maximal set of linearly independent vectors or a minimal set of vectors that spans the space. The former statement when generalized to modules over associative rings become the concept of Goldie dimension (or uniform dimension). Uniform submodules play a significant role to establish various finite dimension conditions in modules over associative rings. However, certain results can not be generalized to modules over rings unless we impose some assumption(s). In particular, we obtain the relative finite Goldie dimension of the sum of two submodules over a ring if their intersection is a complement submodule. We consider a module over a ring and use the concepts: the relative essential ideal, relative uniform ideal of an A-module M to prove various relative finite dimensional conditions of submodules of M. We provide suitable examples distinguishing the existing notions. We also extend a few results on finite Goldie dimension to the module over nearrings (a natural generalization of a ring wherein only one distributive property is assumed, and addition need not be abelian). We provide few references for basic literature.

5.5 A review on the recent developments in probabilistic normed spaces

Harikrishnan Panackal

The notion of Probabilistic Normed spaces, briefly, PN spaces is a natural consequence of the theory of Probabilistic Metric spaces. These spaces were introduced by Karl Menger (Menger, 1942) by giving the idea of a statistical metric, i.e. of replacing the number $d(p, q)$, which gives the distance between two points p and q in a nonempty set S, by a distribution function ν_{pq} whose value $\nu_{p,q}(t)$ at $t \in]0, +\infty]$ is interpreted as the probability that the distance between the points p and q is smaller than t. The theory was then brought to its present state by Schweizer and by Sklar in a series of papers. In this talk, we establish some properties of invertible operators, convex, balanced, absorbing sets and $\mathcal{D}-$boundedness in Šerstnev spaces. We discuss the analogue of Banach fixed point theorem in PN spaces. We show that some PN spaces (V, ν, τ, τ^*) , which are not Šerstnev spaces, in which the triangle function τ^* is not Archimedean can be endowed with a structure of a topological vector space. Finally, we illustrate that the topological spaces obtained in such a manner are normable under certain given conditions.

5.6 On the Leavitt Path Algebras of a Class of Random Graphs

Guadalupe Rafael Molina Rincon

Let G be a finite multigraph, and $\mathcal{O}G$ its associated oriented line graph, as defined by Kotani and Sunada. The Leavitt path algebra of G is $L_{\mathbb{K}}(\mathcal{O}G)$. In this talk, I will report a number of results that concern some asymptotic properties of the Leavitt path algebras associated with a class of random graphs.

5.7 The algebraic loop of relativistic velocities refined

(A guide for spacetime travelers)

Jerzy Kocik

The algebra of the relativistic composition of velocities is shown to be isomorphic to an algebraic loop defined on division algebras (including quaternions, and octonions). This makes calculations in special relativity particularly effortless and straightforward. It also resolves the problem of the nonassociativity of the velocity composition. The algebraic elegance brings about an additional value.

Session 6

Homology, Dualities, Manifolds

6.1 Skein modules and algebras of links in 3-manifolds

Józef H. Przytycki

Theory of skein modules perfectly fit into broad Idea which Zbyszek Oziewicz was developing: Applied Category Theory.

I introduced skein modules in April of 1987 with the goal of building an algebraic topology based on knots. The main object used in the theory is called a skein module which is associated to any arbitrary 3-manifold. There are several skein modules which can be associated to a 3-manifold each of which captures some information of the manifold based on the knot theory that the manifold supports. What all these modules have in common is that they generalize the skein theory of the various link polynomials in S3, for example, the Alexander, Jones, Kauffman bracket, HOMFLYPT, and Kauffman 2-variable polynomial link invariants. Thus, the theory of skein modules, can be seen as the natural extension of quatum link invariants in S3 to arbitrary 3-manifolds. In an interesting twist Edward Witten became in- terested in theory and suggested general conjecture about Kauffman bracket skein modules of closed three manifolds (over the ring of rational function). I will describe its recent solution by Gunningham, Jordan, and Safronov and related torsion conjectures.

6.2 Golden Ratio, variational principles, cyclic and wave phenomena, quanta

Giovanni P. Gregori

Natural science relies on a few basic clues and hunches. Our cognitive approach must take into account the limited available observational information (this is the empirical constraint; we know laws compatibly with available observations). Variational principles (macro-approach) are better suited than microdescriptions, as the knowledge is not requested of the ultimate fundamental laws. In contrast, the focus is only on the observed overall “integral” behavior of the system. In addition, a few almost obsessive manifestations of regularity impose some logical constraints in our cognitive process. The Golden Ratio is one of the most diffuse regularities on natural phenomena – in physics, but also in biology and natural sciences, including psychological reactions – although it is presently unexplained. In addition, cyclic features are a premise for wave phenomena. The non-vanishing duration of a cycle imposes a quantized structure of reality in terms of miniquanta, that are a sub-structure of the standard quanta envisaged by spectroscopy. All these items share close mutual logical links - and constitute the framework for an approach to observations much behind the restricted realm of physics.

6.3 Some categories of Alexandroff spaces

Juan Antonio Pérez

In this work some isomorphisms of categories are explored in the aim to understand Alexandroff spaces. Finite spaces are examples of this sort of topological spaces, so are Boolean groups and minimal models. Compactness reveals to be a very remarkable feature for function spaces based on Alexandroff ones. Some applications are exposed in social sciences, such as electoral systems and design patterns for computer programming.

6.4 Parametric Hopf Adjunctions

Adrian Vazquez-Marquez

This article belongs to a series where 2-adjunctions relating adjunctions and monads are applied to classical monad theory. In this installment, parametric Hopf adjunctions are analysed and related to parametric Hopf monads. In order to do so, the definition of adjoint objects is revised along with the definition of classical parametric adjunctions.This article is mainly based upon the ideas laid out in the seminal article of A. Bruguieres, S. Lack and A. Virelizier [1]. In the referred article, the authors characterise Hopf monads in order to lift, in particular, closed monoidal structures. Note that this step is the most logical one after lifting monoidal structures to categories of Eilenberg-Moore algebras [5].In this article, the same approach is taken but their ideas are developed into a 2-categorical framework, cf. [2, 4]. Also the 2-adjunction that relates adjunctions and monads is weaken into a strict quasi adjunction between the 2-categories of adjunctions and monads [3]. This framework enable to work with the concepts of parametric Hopf adjunctions and monads and relate them.References[1] Bruguieres, A.; Virelizier, A. and Lack, S. Hopf Monads on Monoidal Categories. Adv. Math. 227, 2 (2011), 745-800.[2] Climent Vidal, J. and Soliveres Tur, J. Kleisli and Eilenberg-Moore Constructions as Part of a Biadjoint Situation. Extracta Math. 25, 1 (2010), 1-61.[3] Gray, J.W. Formal Category Theory: Adjointness for 2-categories Vol. 391 of Lecture Notes in Math. Springer, 1974.[4] López Hernández, J.L.; Turcio Cuevas, L.J. and Vazquez-Marquez, A. Applications of the Kleisli and Eilenberg-Moore 2-adjunctions. Categories and General

Algebraic Structures with Applications 10, 1 (2019), 117-156.[5] Moerdijk, I. Monads on Tensor Categories. J. Pure Appl. Algebra 168, 2-3 (2002), 189-208.

6.5 Points: Classical Perspectives and Quantum Refraction

Enrique Bojorquez

In this talk we will explore the concept of 'point' and how does it translate to different contexts. We'll begin by seeing points from a categorical language, as morphisms that have the terminal object as their domain. We'll then use Stone Duality and Gelfand Duality to translate the points of Stone and Locally Compact topological spaces to morphisms in the category of Boolean and $C\star$ algebras respectively. Then we'll look at other definitions that are equivalent in the commutative case, such as irreducible representations, pure states, characters, maximal ideals. And finally, some examples to see how these equivalences don't hold in the non-commutative case, particularly in the rotation algebra and group of 2x2 matrices.

Session 7

Lie Group Symmetry, Quantum Group Symmetry

7.1 $SU(n)$ and Quantum $SU(n)$ Symmetries in Physical Systems

Hanna Makaruk

Presence of $SU(n)$ or other Lie group symmetry in a physical system is its powerful, usually underutilized property. In many cases it allows for finding analytical solutions to nonlinear differential equations describing this system. Power of the method is presented on diversified examples from mathematical physics: Lie-group symmetries in finding solutions of generalized, multidimensional theory of gravity; analytical Dirac–equation solutions for description of conducting polymers; stability of qubit states in quantum computers; spatial defects in condensed matter; reconstruction of $3D$ object from its $2D$ tomographic image; numerical solutions stability for Euler equations. The next question after obtaining such Lie group symmetric solution is: does a generalized solution with appropriate quantum Lie group symmetry exists for the given physical system, and if yes what is the physical meaning of the deformation parameter q introduced by such solution. In many cases it can be identified. Any $SU(n)$ solution is by its nature singular, assuming a perfect symmetry of the physical system discussed. Such solution gives a powerful insight to theoretical physics, yet the assumption may be too demanding for experimental applications. Deformation parameter q from a quantum group symmetry allows for a continuum of solutions, more applicable to experiments."

7.2 The Order of Finite Generation of $\mathrm{SO}(3)$ and Optimization of Rotation Sequences

Danail Brezov

The paper provides a generalization to a classical result due to Lowenthal and a more recent one due to Hamada on the order of finite generation of the rotation group SO(3). The novelty here is that it considers decompositions into factors with more than two invariant axes and provides intuitive proofs relying on rather basic geometry, such as incidence relations and triangle inequalities. A simple estimate for the number of factors for a given transformation (in terms of spherical distances) is provided and possible applications are discussed, such as engineering problems in robotics and optimization of rotational sequences, following as well as qubits and gates. We also comment on the noncompact analogue and its relation to hyperbolic geometry and quantum mechanical scattering.

7.3 Path Norms on a Matrix

Varsha, S Aishwarya, Syam Prasad Kuncham and Babushri Srinivas Kedukodi

Let A be an $m \times n$ matrix with real/complex valued entries. We define row path norm and column path norm of A and relate these norms with other standard matrix norms. Brute-force methods to compute these norms are found to have exponential running time. Hence we present $O(N^2)$ algorithms for computing the path norms, where $N = max\{m, n\}$.

7.4 On absorbing ideals of N-groups

Syam Prasad Kuncham, Tapatee Sahoo, Babushri Srinivas Kedukodi and Harikrishnan Panackal

We consider a zero-symmetric right nearring N, a generalization of a ring in which addition need not be abelian and exactly one distributive axiom is assumed. We define (i, 2)-absorbing ideals (where i=0,3,e) of an N-group as generalization of the prime ideals defined in [6]. We provide suitable examples and prove some properties. In [2], the authors studied the interplay between the prime ideals of a nearring and those of matrix nearring. We extend the relations to absorbing ideals and establish the one-to-one correspondence between the absorbing ideals of N and those of matrix nearring over N.

Session 8

Clifford Algebra, Spheroidal Quaternions

8.1 Spheroidal Quaternions

Garret Sobczyk

Prolate and oblate spheroidal coordinates have found many recent applications in theoretical physics, physical chemistry, planetary science and engineering in making precise calculations and predictions where the limiting case of spherical coordinates is not adequate. A prolate ellipsoid is generated by the rotating an ellipse about its principal axis, whereas an oblate ellipsoid is generated by rotating an ellipse about it non-principal axis. Prolate and oblate coordinates are introduced in a unified way in terms of spheroidal quaternions I, J, K having the unusual properties that $dI = J$, $dJ = -I$, and $dK = 0$. We explore the properties of these quaternions, and their relationship to prolate and and oblated coordinates in the geometric (Clifford) algebra of Euclidean space.

8.2 Special affine quaternion domain Fourier transform

Eckhard Hitzer

This paper undertakes a special affine Fourier transformation [S. Abe, J. T. Sheridan, Generalization of the fractional Fourier transformation to an arbitrary linear lossless transformation an operator approach, J. Phys. A: Math. Gen. 27(12), pp. 4179-4187 (1994). S. Abe, J. T. Sheridan, Optical operations on wave functions as the Abelian subgroups of the special affine Fourier transformation, Opt. Lett. 19(22), pp. 1801-1803 (1994)] generalization of the quaterion domain Fourier transform (QDFT) [E. Hitzer, The Quaternion Domain Fourier Transform and its Properties. AACA 26, pp. 969-984 (2016)] that can process signals with both range and domains being subsets of quaternions $\backslash HQ$. The resulting special affine quaternion domain Fourier transform includes a.o. the QDFT itself, and for quaternion domain functions a linear canonical transform, a fractional Fourier transform, a lense transform, a free-space propagation transform and a magnification transform. We establish the following properties of the SAQDFT: computation in terms of the QDFT, quaternion coefficient linearity, the zero quaternionic frequency magnitude limit, continuity, shift and modulation, inversion, Rayleigh (Parseval) energy preservation, and partial and directional derivatives. Finally we establish the directional uncertainty principle for the SAQDFT, as well as the (direction independent) uncertainty principle

8.3 Solutions of Inhomogeneous Generalized Moisil-Teodorescu Systems in Euclidean Space

Juan Bory-Reyes and Marco Antonio Pérez-de la Rosa

Let $\mathbb{R}^{(s)}_{0,m+1}$ be the space of s-vectors $(0 \leq s \leq m+1)$ in the Clifford algebra $\mathbb{R}_{0,m+1}$ constructed over the quadratic vector space $\mathbb{R}^{0,m+1}$, let $r, p, q \in \mathbb{N}$ with $0 \leq r \leq m+1$, $0 \leq p \leq q$ and $r+2q \leq m+1$ and let $\mathbb{R}^{(r,p,q)}_{0,m+1} = \sum_{j=p}^{q} \bigoplus \mathbb{R}^{(r+2j)}_{0,m+1}$. Then a $\mathbb{R}^{(r,p,q)}_{0,m+1}$–valued smooth function F defined in an open subset $\Omega \subset \mathbb{R}^{m+1}$ is said to satisfy the generalized Moisil-Teodorescu system of type (r, p, q) if $\partial_x F = 0$ in Ω, where ∂_x is the Dirac operator in $\mathbb{R}^{m+1}$. To deal with the inhomogeneous generalized Moisil-Teodorescu systems $\partial_x F = G$, with a $\sum_{j=p}^{q} \bigoplus R^{(r+2j-1)}_{0,m+1}$–valued continuous function G as a right–hand side, we embed the systems in an appropriate Clifford analysis setting. Necessary and sufficient conditions for the solvability of inhomogeneous systems are provided and its general solution described.

8.4 Clifford algebras in Quantum Computing

Carlos Efraín Quintero Narvaez and Dalia Cervantes Cabrera

Different applications of Clifford algebras have been made in quantum computation science [1],[2]. An especially useful one, identifies multipartite states and quantum gates as elements of the geometric algebra (Clifford algebra) of a relativistic configuration space called the multipartite spacetime algebra (MTSA) [3].The traditional presentation of them is through complex Hilbert spaces and their tensor products [4],[5]. This geometric algebra approach entails a unifying point of view, thereby the space of states and unitary operators acting on them become united with both being made just by multivectors in real space. The MTSA technique in conjunction with the spin group formalism will allow searching for universalsets of quantum gates [6]. In his work, we give the explicit steps to obtain the above identification from Hilbert spaces to MTSA perspectives. Also we provided illustrations to the action of some usual quantum gates over qubits and bipartita states like rotations.

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8.5 Complete system of solutions for some Maxwell's equations

Pablo Enrique Moreira Galván

The Maxwell's equations for static (time-independent) fields in anonchiral inhomogeneous medium in sourceless conditions is equivalent to quaternionic differential equations. We construct an invertible operator which transmutes monogenic functions into solutions of the Maxwell's equations, due to the completeness of Grigor'ev polynomials our system is also complete, ie, every solution of the Maxwell's equations can be approximated arbitrarily closely on any compact subset by a finite right linear combination of the images of Grigor'ev polynomials under the transmutation operator.

8.6 From Grassmann to Clifford

Arkadiusz Jadczyk

In 1986, in a paper carrying the exact title of the present talk, Zbigniew Oziewicz introduced the concept of a Clifford algebra of a biliniar form with an arbitrary antisymmetric part. He speculated about possible applications of such a construction in physics, in particular to nucleons, quarks, nuclear shell theory, suggesting also that it may be a way to construct Clifford algebras for symplecting forms. Here we outline this idea making use of the powerful algebraic machinery developed already in 1959 in one of the volumes of the Bourbaki treatise. We will also describe recent revelopments in this area, in particular by Ablamowicz, Fauser and Lounesto, and suggests a different physical interpretation.

Session 9

Topological Quantum Field Theories

9.1 Vortices, knots, Chebyshev polynomials, Frobenius algebras, topological quantum field theories

Robert Owczarek

Studying vortices in superfluid helium, I found it necessary to consider knotted and linked vortices, that led to an interesting but not rigorous description of the phase transition in superfluid helium in terms of a 2D Ising model. Seeking rigor, I entered the world of knot theory. In particular, I found an interesting generalization of Kauffman bracket and related skein theory that parallels a generalization of Chebyshev polynomials that seem to play a role as convenient basis in skein algebras as well as being related to certain categorical approaches to representation theory of some Lie groups. Even more, the Kauffman bracket approach is related to Khovanov homology and its various variation and these theories are intimately related to Frobenius algebras (one of favorites of Professor Oziewicz), which can be thought of as topological quantum field theories in 2D. I will give a personal view of some aspects of this story.

9.2 Negative-Energy 4-Spinors and Masses in the Dirac Equation

Valeriy Dvoeglazov

It is easy to check that both algebraic equation $Det(\hat{p}-m)=0$ and $Det(\hat{p}+m)=0$ for $u-$ and $v-$ 4-spinors have solutions with $p_0=\pm E_p=\pm\sqrt{\mathbf{p}^2+m^2}$. The same is true for higher-spin equations. Meanwhile, every book considers the equality $p_0=E_p$ for both $u-$ and $v-$ spinors of the $(1/2,0)\oplus(0,1/2)$ representation only, thus applying the Dirac-Feynman-Stueckelberg procedure for elimination of the negative-energy solutions. The recent Ziino works (and, independently, the articles of several others) show that the Fock space can be doubled. We re-consider this possibility on the quantum field level for both $s = 1/2$ and higher spin particles.

9.3 Yang-Mills Theory in Non-commutative Geometry

Gustavo Amilcar Saldaña

Considering the non-commutative counterpart of some geometrical concepts such as principal bundles, principal connections, associated vector bundles, induced linear connections and hermitian structures, in this talk we will present a formulation of Yang-Mills theory in the framework of Non-commutative Geometry as well as some examples to illustrate it.

9.4 FEYNMAN PROPAGATOR FOR CLOSED TIMELIKE CURVES IN THE KERR METRIC

Miguel Socolovsky

We compute the Feynman propagator associated with closed timelike curves in the neigborhood of the ring singularity in the Kerr metric. The propagator is well defined outside r=0, where it ceases to exist.

Session 10

Quantum mechanics, Leptones, Quarks, Fundamental interactions

10.1 POSSIBLE APPLICATION TO QUANTUM COMPUTERS OF THE EINSTEIN-PODOLSKY-ROSEN ARGUMENT THAT "QUANTUM MECHANICS IS NOT A COMPLETE THEORY"

Ruggero Maria Santilli

In this talk we outline recent verifications of the Einstein-Podolsky-Rosen (EPR) argument that *quantum mechanics is not a complete theory,* with particular reference to the completion of quantum entanglement for point particles into the new notion *EPR entanglement* which represents extended particles in continuous and instantaneous overlapping of their wavepackets thus without any need for superluminal communications. We then suggest for interested colleagues the use of the EPR entanglement for a completion of quantum computers into a more general model that may improve rapidity of calculations, cyber security and energy efficiency. The advance reading of the works listed below is recommended for a full understanding of the lecture.

10.2 Particle Physics viewed from Category Theory

Michael Heather and Nick Rossiter

Zbigniew Oziewicz was always at the cutting-edge of science. As a young scientist he studied particle physics and carried out important work on the properties of the muon. Latterly he pioneered the application of category theory to a diverse range of subjects of topical interest in science and engineering. It is therefore appropriate in memoriam of the late Professor Zbigniew Oziewicz to consider how the properties of the Standard Model of elementary particles in Physics might be viewed from the perception of Category Theory In applied methodology Category Theory is more than just for modelling as in pure mathematics. In postmodern science Category Theory is now the formal language to express metaphysics as promoted formally and informally by the mathematician and philosopher Alfred North Whitehead (1862-1947) in his Process and Reality (1929). The muon is still of great topical interest in that measurement of its magnet moment has recently in April 2021 shed doubt on validity of the Standard Model of Elementary Particles. The early work by Oziewicz on muon capture by a neutron led him into his lifetime work on Operads and more lately into Category Theory. The Feynman Diagram in particle physics is an early informal premonition of category theory. Some tentative work on Feynman Categories by Kaufman and Ward has shown the difficulties in using the category of sets and confirms the results we have found in many applied fields that the logic of set theory is not the natural logic of nature. Newton's World a three or four Euclidean dimensional block has to be replaced by the process of a Topos. The components of a Topos do not need to be independent as the elements of a set. They are individualised but not separable. They are not subject to what Whitehead calls the fallacy of misplaced concreteness. If the Standard Model is to be revised it needs to recognise the features from Category Theory.

10.3 El Modelo Estándar de la física de partículas

Ricardo Gaitán Lozano

El Modelo estándar de la Física de partículas es una teoría cuántica de campos que contiene las simetrías internas del grupo $SU(3)XSU(2)XU(1)$. La teoría contiene el conjunto fundamental de partículas fundamentales: leptones, quarks, bosones de norma y bosones de Higgs. El modelo describe la estructura fundamental de la materia considerando las partículas elementales como entes irreducibles regidos por tres de las cuatro interacciones conocidas (no incluye la gravedad). Hasta la fecha casi todas las pruebas experimentales de las tres fuerzas descritas por el modelo están de acuerdo con sus predicciones. Sin embargo, hay varias problemas que no han sido resueltos en el marco del Modelo Estándar, como las oscilaciones de neutrinos, la materia oscura, entre otros. En esta plática se hará una revisión de algunos aspectos del Modelo Estándar, principalmente el sector electro-débil.

10.4 Dimensiones paralelas

Silverio Joel Sánchez Pérez

Se ha encontrado que investigando la materia oscura encontraron una partícula que podría actuar como un portal hacia la quinta dimensión, la intención original era "explicar el posible origen de las masas de fermiones (partículas) en teorías con una dimensión extra deformada. Esta nueva partícula podría desempeñar un papel importante en la historia cosmológica del universo y podría producir ondas gravitacionales que se pueden buscar con futuros detectores de ondas gravitacionales". Esta partícula pesada, necesariamente conectaría la materia visible que conocemos y que hemos estudiado en detalle con los constituyentes de la materia oscura, asumiendo que la materia oscura está compuesta de fermiones fundamentales, que viven en la dimensión extra, provocando los misterios que rodean a su comportamiento cuando son detectadas desde nuestro universo de cuatro dimensiones.

Session 11

Computability, Models of Computation

11.1 Superdense coding and multiparty communication protocols using quantum systems of several levels

Guillermo Morales-Luna

We consider high-dimensional quantum states which generalise multi-valued logics. Through maximally entangled states, a central part is able to receive $2(n-1)$ information classical bits from n parts by receiving one two-dimensional qubit from each of them. As a second generalisation, we introduce a multi-party protocol for this kind of superdense coding using quantum systems with k quantum levels.

11.2 Approximately computable structures

Valentina Harizanov

In recent years, computability theorists have investigated dense computability of sets, such as generic and coarse computability. These notions of approximate computability have been motivated by asymptotic density problems in combinatorial group theory. We extend the notions from sets to arbitrary countable structures by introducing generically and coarsely computable structures and further notions of densely computable structures. There are two directions in which these notions could potentially trivialize: either all structures in a certain class could have densely computable isomorphic copies, or only those having computable (or computably enumerable) isomorphic copies. We show that some classes of equivalence structures or directed graphs realize each of these extremal conditions, while others realize neither of them. This is joint work with Wesley Calvert and Doug Cenzer.

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11.3 Striking a balance between cumulative knowledge and speed of proofs in classical propositional calculus

Francisco Hernández Quiroz

Searching for highly useful theorems for shortening/speeding proofs in classical propositional calculus makes sense in the context of massive (probably automatic) theorem generation. Useful theorems are those frequently used in shortening other proofs but only as long as the cost of storing/discovering them is less than the cost of proving them anew in a given proof. Accumulation of theorems can become problematic, as theorems pile up fast and the effort put in storing and retrieving them quickly offsets by a wide margin the cost of proving them anew. An interesting goal will be to identify a phase transition in the process of proving theorems when looking for previous theorems to speed a proof stops being worthwhile. Of course, the location of this phase transition will depend on the particular proof system, the proof strategy, and the features of the intended set of theorems. Our previous work suggests this phase transition comes very soon in most cases, and here I present a path to formally prove this conjecture.

11.4 Back to Basics: (co)inductive types and (co)recursion principles

Lourdes Del Carmen González Huesca

In this talk, I will review computational types from the categorical perspective using notions of morphisms and (co)algebras. Such categorical foundations for Computer Science are ground components in functional programming to give utility principles for defining and proving properties. The relation between programming languages and category theory, and even more between these and intuitionistic logic, is sustained by the so-called computational trilogy, a concept briefly addressed here.

11.5 Dynamic rewiring as learning

David Spivak

In this talk I'll discuss interacting dynamical systems whose interaction pattern itself can change in time, based on the states of the systems involved. I'll explain how the categorical abstractions behind this idea fit perfectly with the sort of gradient descent and backpropagation that occurs in deep neural networks.

11.6 Modeling paraconsistent theories of computation in toposes

Francisco N. Martínez-Aviña

In this talk, I show that we can obtain a model of paraconsistent computability theory via the dualization of the internal logic of the effective topos. I begin describing this dualization method, first introduced by Mortensen and Lavers (1995) and further examined and developed by Estrada-González (2015a,b). After applying the method to the case of the effective topos (Hyland 1982), a topos useful for the study of first- and higher-order recursive phenomena, I discuss some conditions to extend this proposal to different toposes. I conclude with some remarks on how to properly compare theories of computation with (radically) different underlying logics, by making a category-theoretic interpretation of Meadows and Weber (2016) discussion on the non-classical foundations of computability theory.

Session 12

Quantum Lie Operations, Geometric Algebra

12.1 Quantum Lie operations as q-operad

Vladislav Khartchenko

According to the Friedrichs criteria, Lie polynomials are characterized as primitive elements of free associative algebra with primitive free generators. Because every Lie polynomial may be considered as a multivariable operation on Lie algebras, this fact yields an idea to define quantum Lie operations as polynomials of the free algebra that are skew-primitive for all slew-primitive values of variables. In line with this idea, a quantum analog of a Lie algebra is the subspace of a Hopf algebra spanned by skew-primitive elements and equipped by quantum Lie operations considered as a q-operad. In the talk, we discuss a necessary and sufficient existence condition, consider the principle n-linear operation and symmetric operations.

12.2 Quantum Hyperbolic Planes

Perla C. Lucio-Peña

We shall explore a new quantum hyperbolic planes theory, generalizing the classical Poincare model. Our basic tool in studying these planes is the theory of quantum principal bundles. We shall consider a *-algebra which can be seen as a cross product between the classical circle algebra, and the quantum hyperbolic plane algebra. If we geometrically interpret B as a quantum principal bundle with the classical circle being its structure group, then it turns out that the quantum hyperbolic plane is the base space for such a bundle. Also we can represent the base of the bundle in an appropriate Hilbert Space of holomorphic functions within the complex unitary disk, so that the coordinate z acts like the multiplication operator. Interestingly, some infinite series of Euler and Ramanujan naturally appear here, indicating that the roots of quantum geometry were already present in the works of great classical masters.

12.3 Weak Gravitational Fields in the Geometric Algebra Approach

David Antonio Pérez Carlos, Augusto Espinoza Garrido, Alejandro Gutiérrez Rodríguez and Zbigniew Oziewicz

The purpose of this paper is to develop the gravitational theory using spacetime geometric algebra. We assume that the reader is familiarized with the concepts used in this powerful tool. We will show how the linear gravitational equations obtained from different approaches, namely, linearized Einstein's field equations (from tensor analysis), the Heaviside gravitational equations (from Gibbs's vector calculus), gravitational equations obtained using quaternions by Arbab, the linear equations from Logunov's relativistic theory of gravitation (all of them, cited below), can be summarized into a single equation, instead of four. Another advantage of using geometric algebra is that all equations are basis free, which means that the system of gravitational equations and its solutions are independent of reference frames.

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